General Linear Modeling in EEGLAB/LIMO

Basic Theory

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The GLM family

T-tests
Simple regression
ANOVA
Multiple regression
General linear model
- Mixed effects/hierarchical
- Timeseries models (e.g., autoregressive)
- Robust regression
- Penalized regression (LASSO, Ridge)

Generalized linear models
- Non-normal errors
- Binary/categorical outcomes (logistic regression)

One-step solution
Iterative solutions (e.g., IWLS)
A regression is a linear model

**Varying factor:** Contrast of image

**Outcome:** Reaction time

A regression is a linear model

- Given an experimental measure $x$ (e.g. contrast)
- We then do the experiment and collect data $RT$ (e.g. reaction time)
- Model: $RT = \beta_0 + x\beta_1 + \epsilon$
- Do some maths / run a software to find $\beta_1$ and $\beta_0$
  \[ ^\wedge \]
- $RT = 23.6 + 2.7x$
A regression is a linear model

For each trial

\[ RT_1 = \beta_0 + 10\beta_1 + \varepsilon_1 \]
\[ RT_2 = \beta_0 + 5\beta_1 + \varepsilon_2 \]
\[ RT_3 = \beta_0 + 7\beta_1 + \varepsilon_3 \]
...

To test for significance compare the original regression model \( RT_i = \beta_0 + c_i\beta_1 + \varepsilon_i \) with the simplified model \( RT_i = \beta_0 + \varepsilon_i \)

Compare the fit

Test if 0 included in confidence interval
An ANOVA is a linear model

**Varying factor:** Type of image

**Outcome:** Reaction time (go/no-go)
\[ RT_{i,j} = \beta_0 + \beta_i + \varepsilon_{i,j} \]

that is to say the data (e.g. RT) = a constant term (grand mean \( \beta_0 \)) + the effect of a treatment (\( \beta_1 \) for fishes, \( \beta_2 \), \( \beta_3 \) for birds and reptiles) and the error term (\( \varepsilon_{i,j} \))

For trial 4 (for example first trial of birds) we have

\[ RT_{2,1} = \beta_0 + 0*\beta_1 + 1*\beta_2 + 0*\beta_3 + \varepsilon_{2,1} \]

For trial 13 (for example second trial of birds) we have

\[ RT_{2,2} = \beta_0 + 0*\beta_1 + 1*\beta_2 + 0*\beta_3 + \varepsilon_{2,2} \]

Statistics: if there is an effect of treatment then error of the simplified model \( RT_{i,j} = \beta_0 + \varepsilon_{i,j} \) should be lower than the original model \( RT_{i,j} = \beta_0 + \beta_i + \varepsilon_{i,j} \)

\[ \hspace{5cm} \text{Compare the fit} \hspace{5cm} \]

This is a GLM that is equivalent to an ANOVA
A GLM can do both a Regression and an ANOVA (ANCOVA)

**Varying factor:** Type of image AND contrast

**Outcome:** Reaction time (go/no-go)

For example, for trial (first bird with contrast $c_{2,1}$) we have

$$ RT_{2,1} = \beta_0 + 0\beta_1 + 1\beta_2 + 0\beta_3 + 0\beta_3 + c_{2,1}\beta_4 + \epsilon_{2,1} $$

Categorical var. ANOVA

Continuous var. REGRESSION
The design matrix

\[ y(1..3) = 1x\beta_1 + 0x\beta_2 + 0x\beta_3 + 0x\beta_4 + c + \text{error} \]
\[ y(4..6) = 0x\beta_1 + 1x\beta_2 + 0x\beta_3 + 0x\beta_4 + c + \text{error} \]
\[ y(7..9) = 0x\beta_1 + 0x\beta_2 + 1x\beta_3 + 0x\beta_4 + c + \text{error} \]
\[ y(10..12) = 0x\beta_1 + 0x\beta_2 + 0x\beta_3 + 1x\beta_4 + c + \text{error} \]

Design matrix
\[ G_1 \ G_2 \ G_3 \ G_4 \ C \]

Matrix: 
\[
\begin{bmatrix}
8 & 1 \\
9 & 1 \\
7 & 1 \\
5 & 2 \\
7 & 2 \\
3 & 2 \\
3 & 3 \\
4 & 3 \\
1 & 3 \\
6 & 4 \\
4 & 4 \\
9 & 4
\end{bmatrix}
\]

\[ Y = D \times \beta + \varepsilon \]

Measures: \( Y \)
Model/Design matrix: \( D \)
Unknown: \( \beta \)
Errors: \( \varepsilon \)
Linear Modeling of EEG data: level 1

Significance: bootstrap trials to get confidence interval of $\beta$s

Electrode difference Between conditions

Categorical var.

Continuous var.

Electrode 1

Central tendency and plots again

Categorical var.

Electrode 1

Continuous var.
Linear Modeling of EEG data: level 1

Electrode 1

Scalp topography of beta difference at a given latency

Electrode 2

Scalp topography of potential difference (masked using beta signif.)

Electrode 3

Limit of the regions masked for significance
1. Interaction design (EEGLAB default)

2. Factorial design

3. Full factorial design
Linear Modeling of EEG data: level 2

Participant 1

channel 1
channel 2
channel 3
channel 4

... Level 2

Standard stats.
2nd level-GLM

GLM: ordinary least square (OLS)
vs. weighted least square (WLS)
Linear Modeling of EEG data: level 2

Level 1

Participant 1

Participant 2

Participant 3

Level 2

2-way ANOVA:
- Main effect 1 (shape)
- Main effect 2 (color)
- Interaction
Linear Modeling of EEG data: level 2

Participant 1

Shape:
Group t-test on $\beta_o - \beta_e$

Color
Group t-test on $\beta_\bullet - \beta_\star$

Interaction
- One sample t-test on $\beta_\bullet, \beta_\star, \beta_\square, \beta_\blacksquare$

Participant 2

Participant 3
Linear Modeling of EEG data: level 2

Mixed effect model
(still a GLM)

Hierarchical GLM

Level 1

Level 2

2-way ANOVA:
- Main effect 1 (shape)
- Main effect 2 (color)
- Interaction

VS